ST. VIVEKANAND MILLENNIUM SCHOOL
HMT Township, Pinjore

TERM:I / OCTOBER, 2025 **CODE: XII/041/Difficult**

Time allowed : 3hours **MM : 80**

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General Instructions :

- **Section A consists of 20 Question and carry 1 marks each.**
- **Section B consists of 5 question and carry 2 marks each.**
- **Section Cconsistsof6 question and carry3markseach.**
- **Section D consists of 4 question and carry5markseach.**
- **Section E consists of 3 question and carry4 marks each. (Case study Question)**

SECTION-A
(Multiple Choice Questions)
Questions1−20 carry1 Mark Each

1. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then value of x + y is

   a. 4 b. 5 c. 6 d. 7

2. If A is square matrix of order 3 and $|A| = k|A|$, then value of k is equal to:

   a.12 b. 4 c. 64 d. None

3. The principal value branch of $\text{cosec}^{-1} x$ is

   a. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ b. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ c. $(0, \pi)$ d. None

4. The function $f: R \to [-1, 1]$ defined by f(x) = cos x, then

   a. both one-one and onto b. not one-one but onto

c. one-one but not onto d. None

5. If A is a matrix of order m × n and B is a matrix such that $AB' and\ B'A$ are both defined, then order of matrix B is

a. m × m b. n × m c. n × n d. $m \times n$

6. If $x = t^2, y = t^3, then\ \frac{d^2y}{dx^2}$ is

a. $\frac{3}{2}$ b. $\frac{3}{4t}$ c. $\frac{3}{2t}$ d. None

7. The relation R = {(a, a), (b, b), (c, c)} on the set {a, b, c} is

a. Symmetric only b. reflexive and transitive only

c. symmetric and reflexive only d. None

8. If the value of a third order determinant is 3 then the value of the square of the determinant formed by the cofactor will be

a. 9 b. 81 c. 27 d. None

9. A ladder, 5 meter long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate 10cm/sec, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 metres from the wall is

a. $\frac{1}{10}\ radian/sec$ b. $\frac{1}{30}\ radian/sec$

c. $\frac{1}{40}\ radian/sec$ d. $\frac{1}{20}\ radian/sec$

10. If $x = a\sin\theta$ and $y = bcos\ \theta, then\ \frac{d^2y}{dx^2}$ is equal to

a. $\frac{b}{a^2}tan^3\theta$ b. $-\frac{b}{a^2}cosec^3\theta$

c. $\frac{b}{a^2}cossec^3\theta$ d. $-\frac{b}{a^2}sec^3\theta$

11. The function f(x) = cot x is discontinuous on the set

a. $[n\pi : n \in Z]$ b. $\{2n\pi : n \in Z\}$

c. $\{2n\pi + 1 : n \in Z\}$ d. None

12. Find $\frac{dy}{dx}$, if $y = \sin^{-1}(3x - 4x^3)$

a. $\frac{3}{\sqrt{1+x^2}}$ b. $\frac{3}{\sqrt{1-x^2}}$ c. $-\frac{2}{1+x^2}$ d. $-\frac{3}{\sqrt{1+x^2}}$

13. Write the number of points where f(x) = $|x + 5| + |x - 3|$ is not differentiable

a. 3 b. 4 c. 5 d. None

14. The value of $\mathbf{cos^{-1}}\left(\boldsymbol{cos}\frac{\mathbf{14\pi}}{\mathbf{3}}\right)$

a. $\frac{14\pi}{3}$ b. $\frac{\pi}{3}$ c. $\frac{2\pi}{3}$ d. $\frac{4\pi}{3}$

15. If A is an invertible of order 3×3 matrix and $|A| = 4$, then value of $|adj(adj\ A)|$ is

a. -8 b. 8 c. 16 d. 256

16. If $A = \begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix} and\ B = \begin{bmatrix}1 & -4\\ 3 & -2\end{bmatrix}$, find $|AB|$

a. 30 b. -70 c. -30 d. None

17. If $\boldsymbol{f(x) = x^3 - 6x^2 + 9x + 3}$ be decreasing function, then x lies in

a. [1, 5] b. (1, 5) c. (1, 3) d. (-1, 1)

18. The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true?

a. a=3, b = 5 b. a= 2, b= 4 c. a=4, b=6 d. a=5, b=3

19. **ASSERTION(A):** The maximum value of the function $f(x) = x^5, x \in [-1, 1]$, is attained at its critical point, x = 0.

REASON(R) : The maximum value of a function can only occur at points where derivative is zero.

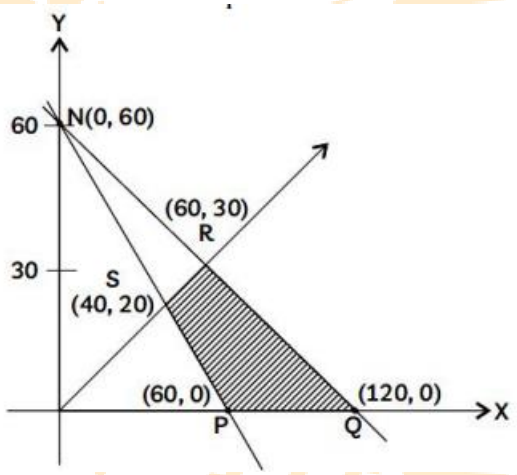
(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true

20. **ASSERTION(A):** The corner points of the bounded feasible region of a LPP are shown below. The maximum value of Z = x + 2y occurs at infinite points.



REASON(R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct

explanation of A.

(c) A is true but R is false.

(d) A is false but R is true

SECTION B
Questions21-25carry2marks Each

21. Show that the relation S in the set R of real numbers, defined as$S = \{(a, b): a, b \in R \ and \ a \leq b^2\}$is neither reflexive, nor symmetric, nor transitive.

22. Find value of k, if $\sin^{-1}\left[k \tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$

23. If $2\begin{bmatrix}3 & 4\\ 5 & x\end{bmatrix} + \begin{bmatrix}1 & y\\ 0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\ 10 & 5\end{bmatrix}$, then find $(x - y)$.

24. If $y = \sin^{-1}(\sqrt{x})$, then find $\frac{dy}{dx}$

25. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is increasing on R.

SECTION C
Question26-31carry3marks each

26. For what value of k is the function$f(x) =$
$\begin{cases}\frac{Sin\ 5x}{3x} + cos\ x & if \ \ x \neq 0\\ k & if \ \ x = 1\end{cases}$ is continuous at x = 0.

27. A man 1.6 m tall walks at the rate of 0.5 m/s away from a lamp post, 8 metres high. Find the rate at which his shadow is increasing and the rate with which the tip of shadow is moving away from the pole.

28. Given $A = \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

29. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

30. If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

31. Find the area bounded by the curve $y = \sin x$ between $x = 0\ and\ x = 2\pi$.

SECTION D
Question32 –35carry5 marks each

32. Show that each of the relation R in the set$A = \{(a, b): 0 \leq x \leq 12\}$, given by R = {(a, b): |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find $A^{-1}. Using\ A^{-1}\ solve\ the\ system\ of\ equations: 2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5, x + y - 2z = -3$

34. Show that the minimum of Z occurs at more than two points. Minimise and Maximise Z = x + 2y subject to x + 2y ≥ 100, 2x – y ≤ 0, 2x + y ≤ 200; x, y ≥ 0.

35. Find the absolute maximum and the absolute minimum value of the function : $f(x) = 4x - \frac{1}{2}x^2,\ x \in [-2, \frac{9}{2}]$

OR

Prove that $y = \frac{4Sin\ x}{(2+\cos x)} - x$is an increasing function of x in $[0, 2\pi]$.

SECTION E
Casestudyquestion(eachquestioncarry4marks)

36. The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V

$$V(t) = \frac{1}{5}t^3 - \frac{2}{5}t^2 + 25t - 2$$

where t represents the time and t = 1, 2, 3.... corresponds to year 2001, 2002, 2003, ... respectively.

Based on the above information, answer the following questions:

(a) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(b) Prove that the function V(t) is an increasing function.

37. Three schools A, B and C decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each respectively. The numbers of articles sold are given as

School /Article	A	B	C
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40



Based on the information given above, answer the following questions:

I. What is the total money (in Rupees) collected by the school A?

II. What is the total amount of money (in ₹) collected by schools B and C?

III. What is the total amount of money collected by

all three schools A, B and C?

OR

If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

38. Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L

Based on the above information, answer the following questions.

(i) Let relation R be defined by $R = \{(L1, L2): L1 \parallel L2$ where $L1, L2 \in L\}$. What is the type of relation R? (1)

(ii) Check whether the function $f : R \rightarrow R$ defined by $f(x) = x - 4$ is bijective or not. (1)

(iii) Let $f : R \rightarrow R$ be defined by $f(x) = x + 4$. Find the range of $f\{x)$. (2)